Half-yearly Examination - 2016-2017
MATHEMATICS

Std. : XII

Time : 3 Hrs. + 15 min
F. M. : 100

Answer question 1, and five other questions from Section A and two questions from either Section B or Section C

All calculations including rough works should be shown as part of the answer.

The intended marks are given in the [ ]

Section A

1 a If $\begin{bmatrix} x^2 & 3 & 4 \\ 1 & 9 & 8 \end{bmatrix} + \begin{bmatrix} -3x & 1 & -5 \\ -3 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 \\ -2 & 7 & 2 \end{bmatrix}$, find the values of x [3]

b Solve $\tan^{-1}\frac{1}{2x+1} + tain^{-1}\frac{1}{4x+1} = \tan^{-1}\frac{2}{x^2}$ [3]

c Prove that 3x-4y=5 is a tangent to the hyperbola $x^2 - 4y^2 = 5$ also find the point of contact [3]

d Evaluate $\lim_{x\to 0} (\cos x)^{1/x^2}$ [3]

e Evaluate: $\int \frac{\sec^2(2\tan^{-1}x)}{1+x^2}dx.$ [3]

f Evaluate $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$ [3]

g The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. if the probability of getting at least one contract is 4/5 what is the probability that he will get both? [3]

h Find the most likely weight of a student corresponding to height 156 cm from the following data: [3]

	Weight (in kg)	*Height (in em)*
Average	65	142
S.D.	2.5	3.5

Correlation coefficient of the weight of students and their height is 0.8.

i) If $y = \sqrt{3x+2}$ prove that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ [3]

j Evaluate : $\int \frac{x^2 dx}{x^2+4x+2}$. [3]

2 a Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$. [5]

solve the equations by matrix method [5]
x + 2y + z = 4 ,-x + y + z = 0 , x - 3y + z = 2

3a Use L M V theorem to find a point on the curve $y = x^3 - 3x$, Where the tangent to the curve is parallel to the chord joining $(1,-2) and (2,2)$ [5]

3b Find the centre, foci, and equation of directrix of the conic [5]
$12x^2 + 4y^2 - 24x + 32y - 20 = 0$. Identify the conic.

4(a) Prove that $\cos^{-1}\frac{4}{5} + \sin^{-1}\frac{12}{13} + \tan^{-1}\frac{63}{16} = \Pi$ [5]

4(b) Draw the circuits represented by the function $(a+b`)(a`+b)(a`+b`)+a'b`..$ [5]
Simplify it and draw the simplified circuit.

5a show that the height of an open cylinder of given surface and greatest volume is equal to the radius of the base [5]

5b If $u = v^3 \log\frac{1}{v}$ show that $v.\frac{d^2u}{dv^2} - 2\frac{du}{dv} + 3v^2 = 0$ [5]

6a Evaluate $\int_0^{\pi} \log\sin x dx$ [5]

6b Find the area enclosed by the parabola $y^2 = 8x$ and the line $y = 4x$. [5]

7a Compute the coefficient of Karl Pearson's correlation coefficient for the following data. And interpret the results. [5]

x	15	22	8	8	7	7	7	6	6	5
y	10	25	17	11	13	17	20	13	9	15

7b Marks obtained by 10 students in Maths and Statistics are given below: [5]
Find the lines of best fit. Estimate the maths mark when the statistics mark is 46

Marks in Maths	79	87	75	73	67	64	39	42	26	35	
Marks in Statistics	73	85	91	67	55	51	55	39	31	44	

8a There are two bags one bag contains 4white and two black balls. Second bag contains 5 white and 4 black balls. 2 balls are transferred from the first bag to the second bag and ,then a ball is taken from the second bag . what is the probability that it is white? [5]

8b A problem in mathematic is given to 3 students. The chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that i) the sum is solved ii) the sum solved by exactly one student iii) the sum is solved by at most 2 student [5]

Section B
(Answer any two questions)

9a In a triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ where a, b, c A ,B and C has the usual meaning [5]

9b Prove that $[\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}] = [\bar{a}\ \bar{b}\ \bar{c}]^2$. [5]

10a Find the vector equation of the line passing through the point (2,3,2) and parallel to the line $\vec{r} = -2\hat{i} + 3\hat{j} + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ and find the distance between these lines [5]

10b Find the equation of the plane through A(-1, 1, 1) and B(4, 2, -11) and perpendicular to the plane $x - 2y + 2z = 3$. [5]

Section C
(Answer any two questions)

11a A bill of exchange for Rs 846.50 at 4 months after sight drawn on 12 January 2008 and accepted on 16th January was discounted at 3.5%on 8th February 2008.find the banker's discount and discounted value of the bill. [5]

11b A firm has the following total cost and demand functions ; [5]

$$C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$$

$$x = 100 - p$$

Find i) marginal cost ii)the profit maximizing the out put ii) Average cost

12a The following table gives the numbers of failures of commercial industries in a country during the years 1975 to 1990.find 4 yearly moving averages and plot the original figures and moving averages in a graph. [5]

Year	1975	1976	1977	1978	1979	1980	1981	1982
Number of failures	23	26	28	32	20	12	12	10
Year	1983	1984	1985	1986	1987	1988	1989	1990
Number of	9	13	11	14	12	9	3	1

12b Taking 1975 as the year with an index number 100, calculate an index number for 1979, based on weighted aggregate method derived from the table given below : (5)

Commodity	A	B	C	D
Weight	30	15	25	30
Price per unit in 1975	20	10	5	40
Price per unit in 1979	24	20	30	40